A Simplified Approach to Evaluate Fragility and Reliability of Buildings Under Bidirectional Seismic Loading

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ABSTRACT

The reliability assessment of buildings subjected to seismic ground motions is often a computing intensive task since an analytical solution is generally unavailable, especially if nonlinear inelastic responses are considered. In the present study, the use of a nonlinear inelastic 2-degree-of-freedom (2DOF) system to approximate a bi-symmetric building is considered. The equivalent 2DOF system is developed based on capacity surface obtained from nonlinear pushover analysis and validated using the results from incremental dynamic analysis. The equivalent model is used as a proxy to the building for the reliability analysis considering bi-directional horizontal ground motions. The overall procedure is illustrated for a designed timber building. Comparison of the reliability of the building subjected uni- and bi-directional is presented and fragility surface in terms of the SA in two horizontal orthogonal direction is given.

Keywords: Reliability, bidirectional ground motion, seismic hazard, nonlinear dynamic analysis.

INTRODUCTION

Reliability analysis of structures subjected to seismic demand is essential part of seismic risk modeling and design code calibration. The commonly used methods to evaluate structural reliability include the first- and second-order reliability methods, response surface method and simulation techniques [1, 2]. The first- and second-order reliability methods are efficient but are inadequate for a class of structural reliability analysis problem, where the derivatives of the considered limit state function are discontinuous. The application of simulation techniques to estimate the structural reliability could lead to accurate results but computing intensive task if the evaluation of the limit state function involves the time history analysis of a structure modeled using complex 3D nonlinear inelastic finite element model. The use of the response surface method can be efficient and adequate, especially if the response surface provides a good fit to the actual system behaviour near the design point [1], which is unknown a priori.

To overcome some of the mentioned problems and to gain efficiency, the reliability analysis of structures subjected to seismic ground motions is carried out by decoupling the analysis of the seismic hazard assessment and the structural capacity [3-5]. In general, a structure is frequently modeled as 2D structural model and the structural capacity subjected to seismic loading is obtained using the nonlinear static pushover analysis or incremental dynamic analysis. The capacity curve is then expressed in terms of structural displacement versus ground motion measure such as spectral acceleration (SA) [5] or in terms of structural displacement (or normalized displacement or ductility demand) versus base shear [6, 7]. However, it seems that an extension of these approaches to 3D structural models under bidirectional horizontal excitations has not been elaborated in the literature. Moreover, the consideration of the 3D structural model subjected to bidirectional horizontal orthogonal ground motions can be important since both horizontal record components affect the elastic and inelastic displacements [8, 9].

The main objective of this study is to provide a simple procedure to estimate reliability and fragility of 3D bisymmetric buildings subjected to bidirectional orthogonal ground motions. The procedure consists of estimating the capacity surface of the structure, developing equivalent nonlinear inelastic 2-degree-of-freedom (2DOF) system, and estimating the structural reliability using the equivalent system. The procedure is illustrated for a designed timber structure, the estimated reliability by considering uni- and bi-directional ground motions is compared, and the fragility surface is developed in terms of SA in two horizontal orthogonal directions.

PROPOSED PROCEDURE

Consider a structure with footprint shown in Figure 1. If the structure is subjected to unidirectional ground motions, the obtained capacity curve is schematically illustrated in the same figure. Note that the capacity curve obtained by nonlinear static pushover analysis does not include the record-to-record variability, while the capacity curves obtained by using the incremental dynamic analysis include the record-to-record variability, but the computation is much more involved.

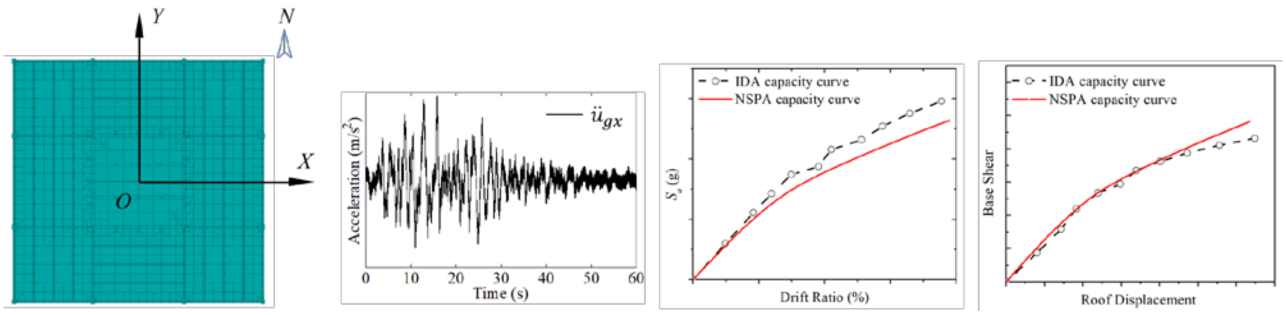


Figure 1. Footprint of a symmetric structure and schematic representation of capacity curve for unidirectional ground motions (the capacity curve is expressed in terms of SA versus drift ratio or Base shear versus roof displacement).

If the structure is subjected to bi-directional ground motions (Figure 2a), the top displacement of the structure associated with the capacity curve projected on the horizontal plane is schematically illustrated in Figure 2b and the obtained capacity is represented by the capacity surface as illustrated in Figure 2c. Figure 2c represents the mean surface that includes the uncertainty in the structural properties and geometric variables. The surface can be obtained based on nonlinear static pushover analysis or the incremental dynamic analysis. If the former is considered, the obtained results does not include the record-to-record variability.

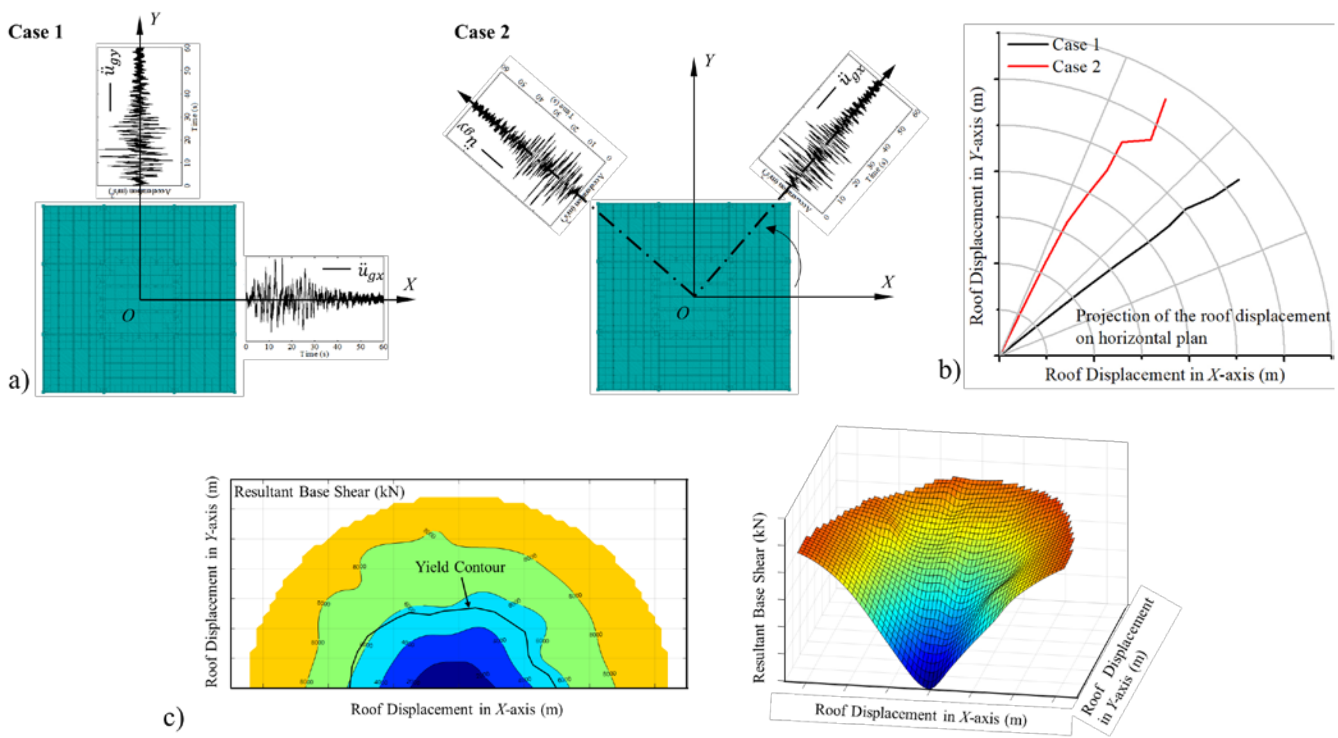


Figure 2. Illustration of capacity surface for a structure subjected to bidirectional ground motion: a) illustration of bidirectional ground motion, b) projection of the structural top displacement on the horizontal surface, c) Contour plot of capacity surface in terms of base shear, d) Surface plot of the capacity (capacity surface).

The analysis procedure used to develop the capacity surface, equivalent 2DOF system, reliability analysis, and fragility assessment are described in the following.

The ***first step*** for the proposed procedure is to carry out analysis using the nonlinear static pushover analysis or incremental dynamic analysis to determine the probabilistic characteristic of the capacity surface illustrated in Figure 2. In the ***second step***, a nonlinear inelastic 2DOF system or 3DOF system is used to mimic the obtained capacity surface of the structure. In the present study, the nonlinear inelastic 2DOF system with Bouc-Wen hysteretic behaviour is adopted for such a purpose. The equations for this system are [3, 8, 10],

$$m\ddot{u}_x + c_x\dot{u}_x + \alpha k_x u_x + (1-\alpha)k_x z_x = -m\ddot{u}_{gx} \tag{1}$$

and,

$$m\ddot{u}_y + c_y\dot{u}_y + \alpha k_y u_y + (1-\alpha)k_y z_y = -m\ddot{u}_{gy} \tag{2}$$

where *u* denotes the displacement; *u* with one and two dots represent the velocity and acceleration; $\ddot{u}_g$ is the horizontal ground motion; *z* is the hysteretic displacement; *m* is the mass, *c* is the viscous damping coefficient, *k* is the stiffness; α is the ratio of post-yield stiffness to initial stiffness; and the subscripts *x* and *y* denote the quantities along the *X*- and *Y*-axes, respectively. The hysteretic displacements z_x and z_y are governed by the following equations [8, 10-13],

$$\dot{z}_x = [\dot{u}_x - \nu z_x I]/\eta \tag{3}$$

and,

$$\dot{z}_y = [\dot{u}_y - \nu z_y I]/\eta \tag{4}$$

where

$$I = |\dot{u}_x||z_x|^{n-1}[\beta + \gamma\,\mathrm{sgn}(\dot{u}_x z_x)] + \frac{\Delta_x^{\,n}}{\Delta_y^{\,n}}|\dot{u}_y||z_y|^{n-1}[\beta + \gamma\,\mathrm{sgn}(\dot{u}_y z_y)] \tag{5}$$

in which Δ_x and Δ_y are the yield displacements along the *X*-axis and *Y*-axis; β, γ, and *n* are shape parameters; η and ν are the parameters related to the degradation, which can be calculated by using,

$$\eta = 1 + \delta_\eta E_{n,b} \tag{6}$$

and,

$$\nu = 1 + \delta_\nu E_{n,b} \tag{7}$$

in which δ_η and δ_ν are the parameters controlling the stiffness degradation and strength degradation, respectively; $E_{n,b}$ represents the normalized dissipated hysteretic energy for biaxial response (see Lee and Hong 2010).

By neglecting the degradation in the strength and stiffness, the capacity surface is then used to determine α and *n*. The symbols *m*, *c* and *k* with subscript shown on the left side in Eqs. (1) and (2) are replaced or represent the equivalent quantities. *m* on the right side of these equations is replaced by effective modal mass, l_x, and l_y respectively.

The ***third step*** is to incorporate the design consideration in the equivalent system so the ductility demand can be evaluated. In such a case, the load applied on the left side of Eqs. (1) and (2) are replaced by

$$m\ddot{u}_{gx} -> l_x\ddot{u}_{gx} = m\frac{\ddot{u}_{gx}}{\zeta_x d_x}\Delta_x \tag{8}$$

and,

$$m\ddot{u}_{gy} -> l_y \ddot{u}_{gy} = m \frac{\ddot{u}_{gy}}{\zeta_y d_y} \Delta_y \tag{9}$$

where d_x and d_y are the $\ddot{u}_{gx}$ and $\ddot{u}_{gy}$ (i.e., earthquake) induced displacements in the corresponding linear elastic system along the X- and Y-axes, respectively; $\zeta_x = \frac{R_{nx}}{R_0 R_d} \frac{(m/l_x)}{L_x}$ and $\zeta_y = \frac{R_{ny}}{R_0 R_d} \frac{(m/l_y)}{L_y}$, in which R_{nx} is R_{ny} are the overstrenthening factor along the X- and Y-axes; R_0 and R_d are overstrengthening and ductility related reduction factors; $L_x = S_{Ax}(T_{nx}, \xi_x) / S_{2475}(T_{nx}, \xi_x)$ and $L_y = S_{Ay}(T_{ny}, \xi_y) / S_{2475}(T_{ny}, \xi_y)$; $S_{2475}(T_{nx}, \xi_x)$ and $S_{2475}(T_{ny}, \xi_y)$ denote the 2475-year return period value of SA (i.e., design SA) along the X- and Y-axes; and S_{Ax} and S_{Ay} are the SA along the X- and Y-axes. Therefore, given the ground motion record, the ductility demand $|u_x / \Delta_x|$ and $|u_y / \Delta_y|$ can be evaluated by solving Eqs. (1) and (2) with the inertial force defined in Eqs. (8) and (9). It can be shown that the ductility demand subjected to the bi-directional ground motion, $\mu_{b,\max}$, can be written as (Lee and Hong 2010),

$$\mu_{b,\max} = \max_{\text{for all } t} \left(|u_x / \Delta_x|^n + |u_y / \Delta_y|^n \right)^{1/n} \tag{10}$$

The ***fourth step*** is to estimate the fragility curve and reliability of the structure (or conditional probability of failure). The estimation of reliability requires the knowledge of the probabilistic characterization of the ground motion measure at the structural site. Consider that the ground motions can be adequately represented by the spectral acceleration (SA) for a random orientation, $S_A(T_n, \xi)$, where T_n is the natural vibration period and ξ is the damping ratio. $S_A(T_n, \xi)$ can be assumed to be lognormally distributed at least at the distribution tail region [14].

Consider that SA along X- and Y-axes, denoted as $S_{Ax}(T_{nx}, \xi_x)$ and $S_{Ay}(T_{ny}, \xi_y)$ are independent and identically distributed as $S_A(T_{nx}, \xi_x)$ and $S_A(T_{ny}, \xi_y)$. If R_{nx} is lognormally distributed with mean $m_{R_{nx}}$ and cov $v_{R_{nx}}$, R_{ny} is lognormally distributed with mean $m_{R_{ny}}$ and cov $v_{R_{ny}}$, $\ln(\zeta_x)$ and $\ln(\zeta_y)$ are normally distributed. Their means $m_{\ln(\zeta_x)}$ and $m_{\ln(\zeta_y)}$, and standard deviations denoted as $\sigma_{\ln(\zeta_x)}$ and $\sigma_{\ln(\zeta_y)}$ are given by,

$$m_{\ln(\zeta_x)} = \ln\left(\frac{m_{R_{nx}}}{\sqrt{1+v_{R_{nx}}^2}} \right) + \ln\left(\frac{L_{mx}}{R_o R_d} \right) - \ln\left(\frac{m_{L_x}}{\sqrt{1+v_{sx}^2}} \right), \qquad m_{\ln(\zeta_y)} = \ln\left(\frac{m_{R_{ny}}}{\sqrt{1+v_{R_{ny}}^2}} \right) + \ln\left(\frac{L_{my}}{R_o R_d} \right) - \ln\left(\frac{m_{L_y}}{\sqrt{1+v_{sy}^2}} \right) \tag{11}$$

and,

$$\sigma_{\ln(\zeta_x)} = \sqrt{\ln\left(1+v_{R_{nx}}^2\right) + \ln\left(1+v_{L_x}^2\right)}, \qquad \sigma_{\ln(\zeta_y)} = \sqrt{\ln\left(1+v_{R_{ny}}^2\right) + \ln\left(1+v_{L_y}^2\right)} \tag{12}$$

where $m_{L_x} = \sqrt{1+v_{sx}^2} \exp\left(-\beta_T \sqrt{\ln\left(1+v_{sx}^2\right)} \right)$, $m_{Ly} = \sqrt{1+v_{sy}^2} \exp\left(-\beta_T \sqrt{\ln\left(1+v_{sy}^2\right)} \right)$, $\beta_T = \Phi^{-1}(1-1/2475)$, $\Phi^{-1}(\bullet)$ is the inverse standard normal distribution function; v_{sx} denotes the cov of $S_{Ax}(T_{nx}, \xi_x)$; and v_{sy} denotes the cov of $S_{Ay}(T_{ny}, \xi_y)$. For example, if the construction is placed in Vancouver, the estimated v_{sx} and v_{sy} based on the results reported in Hong et al. (2006) are 2.29 and 2.35, respectively. The probabilistic models completely characterize ζ_x and ζ_y.

Given the values of ζ_x and ζ_y, and ground motion records, the ductility demand for the system can be determined as mentioned in third step. By employing the simple simulation technique, the probability of incipient yield (or damage) of the system subjected to bidirectional seismic excitations, $P_{D,b} = \Pr\text{ob}(\mu_{b,\max} > 1)$, and the probability of incipient collapse, $P_{C,b} = \Pr\text{ob}(\mu_{b,\max} / \mu_{cap} > 1)$, can be determined, where $\mu_{cap} = \left(\mu_{x,cap}{}^n + \mu_{y,cap}{}^n \right)^{1/n}$; $\mu_{x,cap}$ and $\mu_{y,cap}$ are the ductility capacities of the structure along the X-axis and Y-axis, respectively. $\mu_{x,cap}$ and $\mu_{y,cap}$ could be assumed to be lognormal variates with the mean and cov of $\mu_{x,cap}$, denoted by $m_{\mu_{x,cap}}$ and $v_{\mu_{x,cap}}$, and the mean and cov of $\mu_{y,cap}$, denoted by $m_{\mu_{y,cap}}$ and $v_{\mu_{y,cap}}$, respectively.

For the evaluation of the fragility curve (i.e., the failure probability conditioned on the assigned values of $S_{Ax}(T_{nx},\xi_x)$ and $S_{Ay}(T_{ny},\xi_y)$), the procedure to evaluate the reliability described in the previous paragraph can be used except that v_{sx} and v_{sy} are considered to be equal to zero, and m_{L_x} and m_{Ly} are replaced by (the assigned values of) $S_{Ax}(T_{nx},\xi_x)/S_{2475}(T_{nx},\xi_x)$ and $S_{Ay}(T_{ny},\xi_y)/S_{2475}(T_{ny},\xi_y)$, respectively.

APPLICATION

Description of a design timber structure

The design of the wood building includes the consideration of appropriate design methodology and common practice in structural engineering and architecture. Designed 10-storey mass timber building with footprint of 24 m × 23.2 m shown in Figure 3 is considered in the following. Details of the design can be found in [15]. Basically, it is considered that the first storey height is 4.4 m, and the upper stories has a height of 3.2 m; the cross laminated timber panels are used for floors, roof, shear walls, elevator shaft; the glulam is used for beams and columns. Finite element model of the designed building is developed for the designed wood building by considering the nonlinear behaviour of connectors among the panels. The developed model is also shown in Figure 3. A free vibration analysis is carried out and the first two vibration modes are shown in Figure 4.

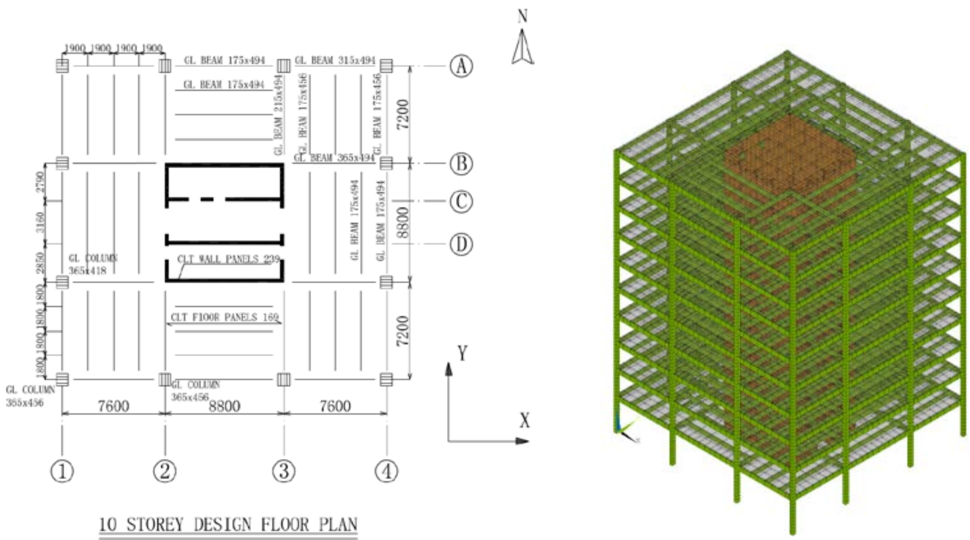
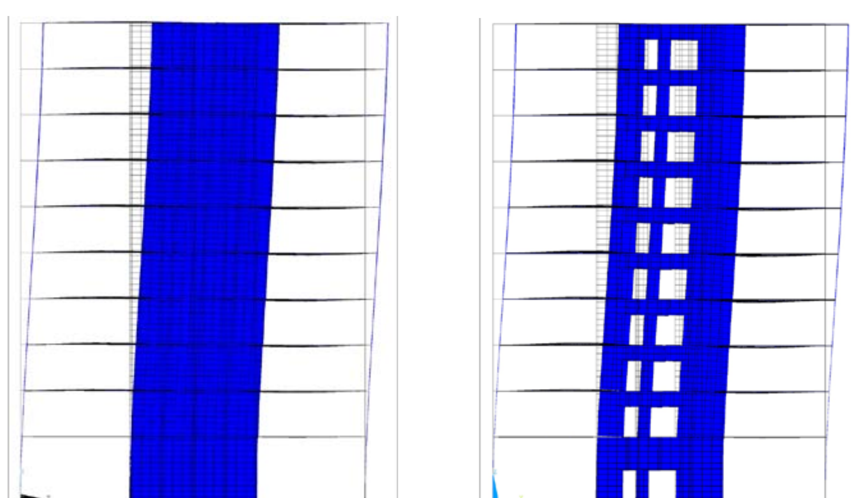


Figure 3. Designed structure and finite element model (Yang et al. 2018).



Sway in *Y*-axis, corresponding period 1.629 s   Sway in *X*-axis, corresponding period 1.301 s

Figure 4. First two sway vibration modes.

Capacity surface

Nonlinear static pushover analysis is carried out by assuming that the load profile along height can be defined based on the sway vibration modes shown in Figure 4. The capacity curve obtained are shown in Figure 5. The use of the nonlinear static pushover analysis is efficient. Details of using the nonlinear static pushover analysis to assess capacity surface (i.e., considering different incidence angle) was discussed in [16]. To validate the adequacy of the obtained capacity surface, the nonlinear incremental dynamic analysis is also carried out and the mean of the capacity curves obtained by considering 11 ground motion records from 11 California earthquake events with moment magnitude ranging from 6.2 to 7.3 is also presented in Figure 5. Comparison of the results of the capacity surfaces obtained by using the nonlinear static pushover analysis and the mean of the capacity surfaces obtained by using the incremental dynamic analysis shown in Figure 5 indicates that the former adequately represents the latter.

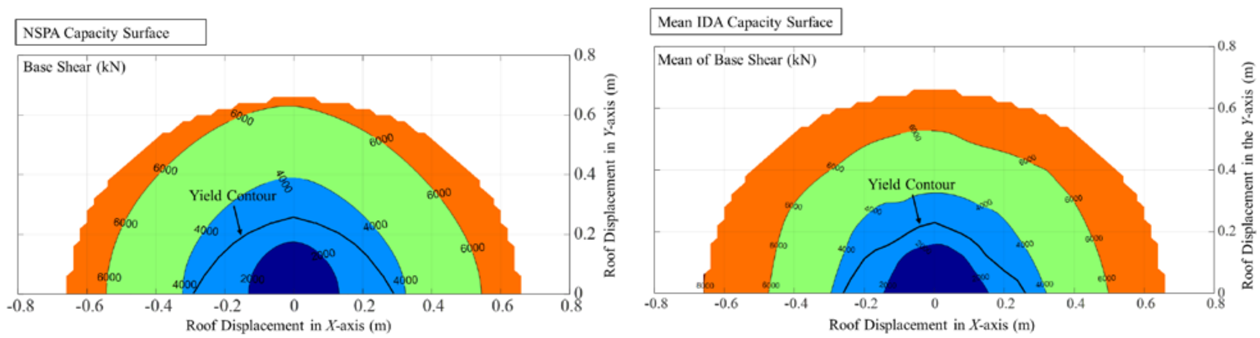


Figure 5. Capacity surface obtained by using the nonlinear static pushover analysis and the nonlinear incremental dynamic analysis.

Ductility demand and reliability

For the reliability analysis and the fragility surface assessment the parameters shown in Table 1 are employed. These parameters are obtained based on the design considerations, and the coefficient of variation values are discussed in the previous section.

Following the analysis procedure described in the previous section, the obtained reliability by considering incipient collapse is shown in Figure 6 for a range of mean ductility capacity values. As can be seen from the figure, the consideration of bidirectional ground motions leads to increased failure probability (i.e., decreased reliability). The ratio of failure probability subjected to bidirectional ground motions to that subjected to unidirectional ground motions ranges from 2 to 7.

Table 1. Parameters used to calculate the reliability along *X*-axis and *Y*-axis.

Parameters	*X*-axis	*Y*-axis
T_n (s)	1.30	1.63
Seismic design load (kN)	1932	1572
L_m	1.81	2.05
Mean of R_n, m_{R_n}	1.87	1.85
cov of R_n, v_{R_n}	0.23	0.28
Mean of $\mu_{x,cap}$ and $\mu_{y,cap}$, $m_{\mu_{x,cap}}$ and $m_{\mu_{y,cap}}$ for failure probability	2.0 to 4.0	
$m_{\mu_{x,cap}}$ and $m_{\mu_{y,cap}}$ for fragility	3.0	
cov of $\mu_{x,cap}$ and $\mu_{y,cap}$, $v_{\mu_{x,cap}}$ and $v_{\mu_{y,cap}}$	0.3	

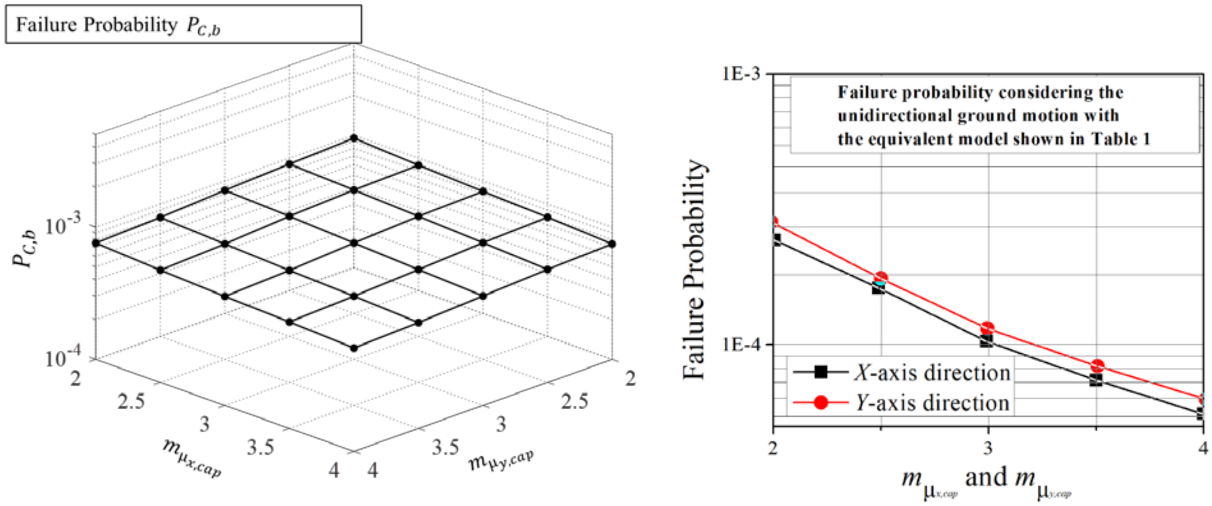


Figure 6. Estimated failure probability $P_{C,b}$ .and failure probability considering the unidirectional ground motion with the equivalent model shown in Table 1

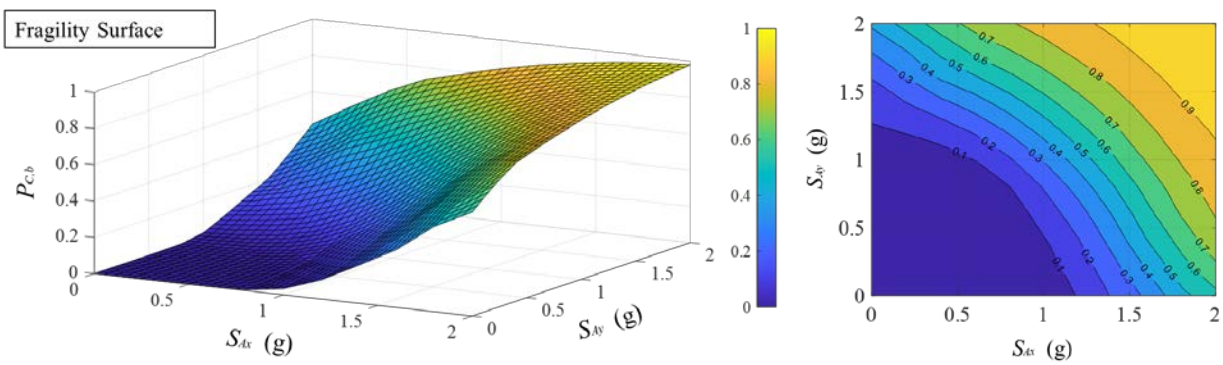


Figure 7. Estimated fragility surface

CONCLUSIONS

A procedure is proposed to assess the reliability and fragility surface of bisymmetrical buildings subjected to bidirectional ground motions. The procedure is simple, it consists of developing the capacity surface, approximating the capacity surface using a nonlinear inelastic 2-degree-of-freedom (2DOF) system, and carrying out the probabilistic analysis for the equivalent 2DOF system.

The estimated failure probability indicates that the consideration of unidirectional ground motions could lead to underestimation of the failure probability. The ratio of failure probability subjected to bidirectional ground motions to that subjected to unidirectional ground motions ranges from 2 to 7. In addition, fragility curves are developed by considering bidirectional ground motions. It is expected that such a simple procedure can be incorporated in the assessment of seismic risk of buildings under bidirectional ground motions.

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